

Limits(सीमा)

Limit :- We say that $\lim_{x \rightarrow a} f(x) = l$ if whenever

$$x \rightarrow a, f(x) \rightarrow l.$$

Fundamental Theorems on Limits

- $\lim_{x \rightarrow a} \{f(x) + g(x)\} = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} \{f(x) - g(x)\} = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} \{f(x) \cdot g(x)\} = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}, \lim_{x \rightarrow a} g(x) \neq 0$
- $\lim_{x \rightarrow a} \{c \cdot f(x)\} = c \cdot \lim_{x \rightarrow a} f(x), [c = \text{constant}]$
- $\lim_{x \rightarrow a} c = c$
- If $f(x) \leq g(x)$ for all x then $\lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} g(x)$
सभी x के लिए यदि $f(x) \leq g(x)$ है तो

$$\lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} g(x)$$

Remember(याद करें)

- $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n a^{n-1}, \text{ where } a > 0$
- $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a$
- $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$
- $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$
- $\lim_{x \rightarrow 0} (1+x)^{1/x} = e$
- $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$
- $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{x}{\tan x} = 1$
- $\lim_{x \rightarrow 0} \cos x = 1$
- $\lim_{x \rightarrow 0} \sin x = 0$

Multiple Choice Questions
(बहु विकल्पीय प्रश्न)

- The value of $\lim_{x \rightarrow 0} 3x$ is
 $\lim_{x \rightarrow 0} 3x$ का मान है

- | | |
|-------|-------------------|
| (a) 3 | (b) $\frac{1}{3}$ |
| (c) 1 | (d) 0 |

- The value of $\lim_{x \rightarrow 2} (3x+5)$ is

- | | |
|--------|---------|
| (a) 3 | (b) 5 |
| (c) 11 | (d) -11 |

- The value of $\lim_{r \rightarrow 1} \pi r^2$ is

- | | |
|-----------|-------------------|
| (a) π | (b) 1 |
| (c) 0 | (d) None of these |

- The value of $\lim_{x \rightarrow 0} (2x - 1)$ is

- | | |
|-------|--------|
| (a) 2 | (b) -1 |
| (c) 1 | (d) 0 |

- The value of $\lim_{x \rightarrow 1} (x^2 + 2x + 1)$ is

- | | |
|-------|--------|
| (a) 4 | (b) -4 |
| (c) 1 | (d) 2 |

- The value of $\lim_{x \rightarrow 3} (x^2 - 9)$ is

- | | |
|-------|--------|
| (a) 9 | (b) 0 |
| (c) 4 | (d) -9 |

- The value of $\lim_{x \rightarrow 0} \frac{x+5}{x-1}$ is

- | | |
|--------|-------|
| (a) -5 | (b) 3 |
| (c) 5 | (d) 0 |

- The value of $\lim_{x \rightarrow -2} (x^2 - 4x)$ is

- | | |
|---------|--------|
| (a) -4 | (b) 12 |
| (c) -12 | (d) 4 |

- The value of $\lim_{x \rightarrow 1} (x+1)$ is

- | | |
|-------|-------|
| (a) 1 | (b) 2 |
| (c) 3 | (d) 4 |

- The value of $\lim_{x \rightarrow 5} (2 - x^2)$ is

- | | |
|---------|--------|
| (a) 23 | (b) 20 |
| (c) -23 | (d) 12 |

- $\lim_{x \rightarrow 0} \log(1-x)$ is equal to

- | | |
|-------------------|-------------------|
| (a) 0 | (b) 1 |
| (c) $\frac{1}{2}$ | (d) None of these |

- $\lim_{x \rightarrow 5} \log(2+x)$ is equal to

- | | |
|--------------|--------------|
| (a) 0 | (b) $\log 2$ |
| (c) $\log 5$ | (d) $\log 7$ |

- $\lim_{x \rightarrow 0} (e^x + 2)$ is equal to

- | | |
|-------|-------|
| (a) 0 | (b) 2 |
| (c) 3 | (d) 1 |

14. $\lim_{x \rightarrow 0} (e^x + e^{2x} + 2x)$ is equal to

- (a) 0 (b) 2
(c) 3 (d) 1

15. $\lim_{x \rightarrow 0} (1 + x + x^2 + x^3 + x^4)$ is equal to

- (a) 0 (b) 1
(c) 5 (d) 4

16. $\lim_{x \rightarrow -1} (1 + x + x^2 + \dots + x^{10})$ is equal to

- (a) 0 (b) 1
(c) -1 (d) 2

17. If $f(x) = \frac{x+2}{x+1}$, $x \neq -1$ then $\lim_{x \rightarrow 0} f(x)$ is

यदि $f(x) = \frac{x+2}{x+1}$, $x \neq -1$ तो $\lim_{x \rightarrow 0} f(x)$ का मान है

- (a) 1 (b) 2
(c) -2 (d) 3

18. If $f(x) = e^{2x} - 8$ then $\lim_{x \rightarrow 0} f(x)$ is

- (a) -8 (b) -7
(c) 8 (d) 7

19. If $f(x) = \log(x+1) + 2$, then $\lim_{x \rightarrow 0} f(x)$ is

- (a) 3 (b) 2
(c) 0 (d) 1

20. The value of $\lim_{x \rightarrow 3} \left(\frac{x^2 + 9}{x + 3} \right)$ is

- (a) 18 (b) 6
(c) 3 (d) 2

21. The value of $\lim_{x \rightarrow 2} \left(\frac{x^2 - 4}{x - 2} \right)$ is

- (a) 4 (b) 2
(c) 8 (d) 0

22. The value of $\lim_{x \rightarrow 3} \left(\frac{x^4 - 81}{x - 3} \right)$ is

- (a) 108 (b) 100
(c) 81 (d) 0

23. The value of $\lim_{x \rightarrow a} \left(\frac{x^{5/2} - a^{5/2}}{x - a} \right)$ is

- (a) $\frac{5}{2}$ (b) $\frac{5a^{3/2}}{2}$
(c) $\frac{2a^{3/2}}{5}$ (d) $a^{3/2}$

24. $\lim_{x \rightarrow 4} \left(\frac{x^3 - 64}{x^2 - 16} \right)$ is equal to

- (a) 6 (b) -6
(c) 0 (d) 2

25. $\lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x} \right)$ is equal to

- (a) -1 (b) 0
(c) 1 (d) None of these

26. The value of $\lim_{x \rightarrow 0} \left(\frac{e^{2x} - 1}{x} \right)$ is

- (a) 1 (b) 2
(c) 0 (d) 3

27. The value of $\lim_{x \rightarrow 0} \left(\frac{e^{5x} - 1}{10x} \right)$ is

- (a) 5 (b) $\frac{1}{2}$
(c) 2 (d) $\frac{1}{5}$

28. The value of $\lim_{x \rightarrow 0} \left(\frac{e^{mx} - 1}{nx} \right)$ is

- (a) $\frac{m}{n}$ (b) m
(c) n (d) $\frac{1}{m}$

29. The value of $\lim_{x \rightarrow 0} \left(\frac{e^{3x} - e^{2x}}{x} \right)$ is

- (a) 1 (b) 3
(c) 5 (d) 0

30. The value of $\lim_{x \rightarrow 0} \left(\frac{e^{mx} - e^{nx}}{x} \right)$ is

- (a) $m - n$ (b) $m + n$
(c) mn (d) $\frac{m}{n}$

31. $\lim_{x \rightarrow 0} \left(\frac{e^{5x} - e^{2x}}{6x} \right)$ is equal to

- (a) $\frac{1}{2}$ (b) 3
(c) 2 (d) $\frac{1}{3}$

32. $\lim_{x \rightarrow 0} \left(\frac{3^x - 1}{x} \right)$ is equal to

- (a) 3 (b) $\log 3$
(c) 1 (d) $\log 2$

33. $\lim_{x \rightarrow 0} \left(\frac{5^x - 1}{2x} \right)$ is equal to

- (a) $\log 5$ (b) $\frac{1}{2} \log 5$
(c) $\frac{1}{2}$ (d) $\log 2$

34. $\lim_{x \rightarrow 0} \left(\frac{a^x - b^x}{x} \right)$ is equal to

- (a) $\log a + \log b$ (b) $\log a - \log b$
(c) $\log(a - b)$ (d) $\log(a + b)$

35. $\lim_{x \rightarrow 0} \frac{5^x - 2^x}{x}$ is equal to

- (a) $\log 5 + \log 2$ (b) $\log 5 - \log 2$
(c) $\log(3)$ (d) $\log(7)$

36. The value of $\lim_{x \rightarrow 0} \frac{2^x - 1}{2x}$ is

- (a) $\frac{1}{2}$ (b) $\log 2$
 (c) $\frac{1}{2} \log 2$ (d) $-\frac{1}{2} \log 2$
37. The value of $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x}$ is
 (a) 0 (b) 2
 (c) 3 (d) 1
38. The value of $\lim_{x \rightarrow 0} \frac{\log(1+3x)}{x}$ is
 (a) 0 (b) 2
 (c) 3 (d) 1
39. If $f(x) = \frac{\log(1+2x)}{x}$ then $\lim_{x \rightarrow 0} f(x)$ is
 (a) 0 (b) 2
 (c) 3 (d) 1
40. If $f(x) = \frac{\log(1+5x)}{10x}$ then $\lim_{x \rightarrow 0} f(x)$ is
 (a) 5 (b) $\frac{1}{2}$
 (c) 2 (d) 10
41. The value of $\lim_{x \rightarrow 0} \left(\frac{\sin 2x}{x} \right)$ is
 (a) 1 (b) 0
 (c) 2 (d) $\frac{1}{2}$
42. The value of $\lim_{x \rightarrow 0} \left(\frac{\tan 5x}{x} \right)$ is
 (a) 1 (b) 4
 (c) 5 (d) 0
43. If $f(x) = \left(\frac{\sin 2x}{4x} \right)$ then $\lim_{x \rightarrow 0} f(x)$ is
 (a) 2 (b) $\frac{1}{2}$
 (c) 4 (d) 1
44. $\lim_{x \rightarrow 0} \left(\frac{\sin 4x}{2x} \right)$ is equal to
 (a) 2 (b) 4
 (c) 8 (d) 6
45. $\lim_{x \rightarrow 0} \left(\frac{\tan 3x}{6x} \right)$ is equal to
 (a) 2 (b) 3
 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$
46. The value of $\lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sin 2x} \right)$ is
 (a) 4 (b) $\frac{1}{4}$
 (c) $\frac{1}{2}$ (d) 2
47. The value of $\lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx}$ is
- (a) m (b) $m+n$
 (c) $\frac{m}{n}$ (d) mn
48. The value of $\lim_{x \rightarrow 0} \frac{\tan 9x}{3x}$ is
 (a) 9 (b) 6
 (c) 12 (d) 3
49. $\lim_{x \rightarrow 0} \frac{\sin 5x}{\tan 6x}$ is equal to
 (a) $\frac{5}{6}$ (b) $\frac{6}{5}$
 (c) 30 (d) 11
50. $\lim_{x \rightarrow 0} \frac{\tan 8x}{\sin 4x}$ is equal to
 (a) 8 (b) 4
 (c) $\frac{1}{2}$ (d) 2
51. The value of $\lim_{x \rightarrow 0} \frac{\tan 7x}{\tan 14x}$ is
 (a) 7 (b) 14
 (c) $\frac{1}{2}$ (d) 2
52. The value of $\lim_{x \rightarrow 0} (1 + \cos x)$ is
 (a) 2 (b) 1
 (c) 0 (d) None of these
53. The value of $\lim_{x \rightarrow 0} (2 + \sin x)$ is
 (a) 2 (b) 1
 (c) 0 (d) None of these
54. $\lim_{x \rightarrow 0} (\sin x + \cos x)$ is equal to
 (a) 0 (b) 1
 (c) 2 (d) 3
55. $\lim_{x \rightarrow 0} (x + \sin x)$ is equal to
 (a) 0 (b) 1
 (c) 2 (d) 3
56. $\lim_{x \rightarrow 0} \sqrt{x+4}$ is equal to
 (a) 0 (b) 1
 (c) 2 (d) 3
57. $\lim_{x \rightarrow 0} \sqrt{\frac{x+4}{x+1}}$ is equal to
 (a) 2 (b) 1
 (c) 4 (d) 0
58. The value of $\lim_{x \rightarrow 2} \frac{x^2 + 2x + 2}{5x}$ is
 (a) 2 (b) 1
 (c) 4 (d) 0
59. The value of $\lim_{x \rightarrow 0} \frac{\sin^2 mx}{\sin^2 nx}$ is
 (a) $\frac{m^2}{n^2}$ (b) $\frac{m}{n}$
 (c) mn (d) $m^2 + n^2$

60. The value of $\lim_{x \rightarrow 0} \frac{\sin^2 3x}{\sin^2 2x}$ is

(a) $\frac{3}{2}$ (b) $\frac{9}{4}$
(c) 5 (d) 36

61. The value of $\lim_{x \rightarrow 0} \frac{\tan^2 6x}{\tan^2 3x}$ is

(a) 36 (b) 9
(c) 18 (d) 4

62. The value of $\lim_{x \rightarrow 0} \frac{\sin^2 5x}{\tan^2 4x}$ is

(a) 100 (b) $\frac{25}{16}$
(c) $\frac{16}{25}$ (d) $\frac{5}{4}$

63. If $f(x) = \sin x$ then $\lim_{x \rightarrow \frac{\pi}{2}} f(x)$ is

(a) 0 (b) 2
(c) 3 (d) 1

64. $\lim_{x \rightarrow \pi} (\sin x + \cos x)$ is equal to

(a) 1 (b) -1
(c) 0 (d) 2

65. $\lim_{x \rightarrow \frac{\pi}{6}} (\sin x - \cos x)$ is equal to

(a) $\frac{1 - \sqrt{3}}{2}$ (b) $\frac{1 + \sqrt{3}}{2}$
(c) $1 + \sqrt{3}$ (d) $1 - \sqrt{3}$

66. The value of $\lim_{x \rightarrow \frac{\pi}{4}} (\tan x)$ is

(a) -1 (b) 0
(c) 1 (d) None of these

67. The value of $\lim_{x \rightarrow 0} (x \cdot \cot 2x)$ is

(a) 1 (b) -1
(c) $\frac{1}{2}$ (d) 2

68. If $f(x) = \tan x + \cot x$ then $\lim_{x \rightarrow \frac{\pi}{4}} f(x)$ is

(a) 2 (b) 1
(c) 0 (d) -1

69. If $f(x) = \sqrt{x^2 + 4x + 1}$ then $\lim_{x \rightarrow 0} f(x)$ is

(a) 0 (b) 1
(c) -1 (d) 2

70. $\lim_{x \rightarrow 2} \left(\frac{x^2 + 4}{x + 2} \right)$ is equal to

(a) 8 (b) 4
(c) 1 (d) 2

Answer key उत्तरमाला

Multiple Choice Questions (बहु विकल्पीय प्रश्न)

- | | | | | |
|--------|--------|--------|--------|--------|
| (1) d | (2) c | (3) a | (4) b | (5) a |
| (6) b | (7) a | (8) b | (9) b | (10) c |
| (11) a | (12) d | (13) c | (14) b | (15) b |
| (16) b | (17) b | (18) b | (19) b | (20) c |
| (21) a | (22) a | (23) b | (24) a | (25) c |
| (26) b | (27) b | (28) a | (29) a | (30) a |
| (31) a | (32) b | (33) b | (34) b | (35) b |
| (36) c | (37) d | (38) c | (39) b | (40) b |
| (41) c | (42) c | (43) b | (44) a | (45) c |
| (46) d | (47) c | (48) d | (49) a | (50) d |
| (51) c | (52) a | (53) a | (54) b | (55) a |
| (56) c | (57) a | (58) b | (59) a | (60) b |
| (61) d | (62) b | (63) d | (64) b | (65) a |
| (66) c | (67) c | (68) a | (69) b | (70) d |

Very Short Answer Type Questions (अति लघु उत्तरीय प्रश्न)

Evaluate (मान ज्ञात कीजिए)

- $\lim_{x \rightarrow 2} \left(\frac{x^5 - 32}{x^3 - 8} \right)$
- $\lim_{x \rightarrow 2} \left(\frac{3x^2 - x - 10}{x^2 - 4} \right)$
- $\lim_{x \rightarrow \pi} \left(\frac{\sin(\pi - x)}{\pi(\pi - x)} \right)$
- $\lim_{x \rightarrow 0} \left(\frac{\cos 2x - 1}{\cos x - 1} \right)$
- $\lim_{x \rightarrow 0} \left(\frac{(1+x)^n - 1}{x} \right)$
- $\lim_{x \rightarrow 0} \left(\frac{\sqrt{1+3x} - \sqrt{1-3x}}{x} \right)$
- $\lim_{x \rightarrow 0} \left(\frac{\cos x}{\pi - x} \right)$
- $\lim_{x \rightarrow 0} \left(\frac{e^{3x} - 1}{2x} \right)$
- $\lim_{x \rightarrow 0} \left(\frac{x^2 - 2x + 1}{x + 1} \right)$
- $\lim_{x \rightarrow \pi} \left(x - \frac{22}{7} \right)$
- $\lim_{x \rightarrow 3} \left(\frac{x^2 + 9}{x + 3} \right)$

$$12. \lim_{x \rightarrow 0} \left(\frac{\sin 7x}{8x} \right)$$

$$13. \lim_{x \rightarrow 0} \left(\frac{\sin 8x}{\tan 2x} \right)$$

$$14. \lim_{x \rightarrow 0} \left(\frac{x+1}{x-1} \right)$$

$$15. \lim_{x \rightarrow 4} \left(\frac{4x+3}{x-2} \right)$$

$$16. \lim_{x \rightarrow 1} \left(\frac{x^{10}-1}{x-1} \right)$$

$$17. \lim_{x \rightarrow 0} \left(\frac{x^5+2x-8}{x-8} \right)$$

$$18. \lim_{x \rightarrow 0} \left(\frac{ax+b}{cx+1} \right)$$

$$19. \lim_{x \rightarrow 0} \left(\frac{2x+1}{5} \right)$$

$$20. \lim_{x \rightarrow 0} \left(\frac{5^x-1}{x} \right)$$

Very Short Answer Type Questions (अति लघु उत्तरीय प्रश्न)

$$1. \text{ Given, } \lim_{x \rightarrow 2} \left(\frac{x^5-32}{x^3-8} \right), \quad \text{form} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 2} \left(\frac{x^5-2^5}{x^3-2^3} \right)$$

$$= \lim_{x \rightarrow 2} \left(\frac{\frac{x^5-2^5}{x-2}}{\frac{x^3-2^3}{x-2}} \right)$$

$$= \frac{5 \times 2^4}{3 \times 2^2}$$

$$= \frac{5 \times 2^2}{3} = \frac{20}{3} \quad \text{Ans}$$

$$2. \text{ Given,}$$

$$\lim_{x \rightarrow 2} \frac{3x^2-x-10}{x^2-8}$$

$$= \frac{3(2)^2-(2)-10}{(2)^2-8}$$

$$= \frac{12-2-10}{4-8}$$

$$= \frac{12-12}{-4}$$

$$= \frac{0}{-4}$$

$$= 0 \quad \text{Ans}$$

$$3. \text{ Given,}$$

$$\lim_{x \rightarrow \pi} \frac{\sin(\pi-x)}{\pi(\pi-x)}$$

$$\text{Let } \pi-x = h \quad \therefore h \rightarrow 0$$

$$= \lim_{h \rightarrow 0} \frac{\sin h}{\pi h} = \frac{1}{\pi} \lim_{h \rightarrow 0} \frac{\sin h}{h}$$

$$= \frac{1}{\pi} \cdot (1) = \frac{1}{\pi} \quad \text{Ans}$$

$$4. \text{ Given, } \lim_{x \rightarrow 0} \frac{\cos 2x-1}{\cos x-1}, \quad \text{form} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 0} \frac{1-\cos 2x}{1-\cos x}$$

$$= \lim_{x \rightarrow 0} \frac{2\sin^2 x}{2\sin^2 \frac{x}{2}}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin x}{\sin \frac{x}{2}} \right)^2$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \times \frac{x/2}{\sin x/2} \times 2 \right)^2$$

$$= (1 \times 1 \times 2)^2$$

$$= 4 \quad \text{Ans}$$

$$5. \text{ Given, } \lim_{x \rightarrow 0} \left(\frac{(1+x)^n-1}{x} \right), \quad \text{form} = \frac{0}{0}$$

$$\text{let } 1+x = h$$

$$\text{as } x \rightarrow 0 \quad \therefore h \rightarrow 1$$

$$= \lim_{h \rightarrow 1} \frac{h^n-1^n}{h-1} = n \cdot 1^{n-1} = n \quad \text{Ans}$$

$$6. \text{ Given,}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+3x}-\sqrt{1-3x}}{x}, \quad \text{form} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{1+3x}-\sqrt{1-3x}}{x} \times \frac{\sqrt{1+3x}+\sqrt{1-3x}}{\sqrt{1+3x}+\sqrt{1-3x}}$$

$$= \lim_{x \rightarrow 0} \frac{1+3x-1+3x}{x \cdot (\sqrt{1+3x}+\sqrt{1-3x})}$$

$$= \lim_{x \rightarrow 0} \frac{6x}{x \cdot (\sqrt{1+3x}+\sqrt{1-3x})}$$

$$= \lim_{x \rightarrow 0} \frac{6}{(\sqrt{1+3x}+\sqrt{1-3x})}$$

$$= \frac{6}{\sqrt{1+0}+\sqrt{1-0}}$$

$$= \frac{6}{1+1}$$

$$= \frac{6}{2} = 3 \quad \text{Ans}$$

7. Given,

$$\lim_{x \rightarrow 0} \frac{\cos x}{\pi - x} \\ = \frac{\cos 0}{\pi - 0} = \frac{1}{\pi} \quad \text{Ans}$$

8. Given,

$$\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{2x}, \quad \text{form} = \frac{0}{0} \\ = \lim_{x \rightarrow 0} \frac{e^{3x} - 1}{3x} \times \frac{3}{2} \\ = 1 \times \frac{3}{2} = \frac{3}{2} \quad \text{Ans}$$

9. Given,

$$\lim_{x \rightarrow 0} \frac{x^2 - 2x + 1}{x + 1} \\ = \frac{0^2 - 2 \cdot (0) + 1}{0 + 1} = 1 \quad \text{Ans}$$

10. Given,

$$\lim_{x \rightarrow \pi} \left(x - \frac{22}{7} \right) \\ = \pi - \frac{22}{7} \quad \text{Ans}$$

11. Given

$$\lim_{x \rightarrow 3} \frac{x^2 + 9}{x + 3} \\ = \frac{3^2 + 9}{3 + 3} \\ = \frac{9 + 9}{9} \\ = \frac{18}{9} = 2 \quad \text{Ans}$$

12. Given,

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{8x}, \quad \text{form} = \frac{0}{0} \\ = \lim_{x \rightarrow 0} \frac{\sin 7x}{7x} \times \frac{7}{8} \\ = 1 \times \frac{7}{8} = \frac{7}{8} \quad \text{Ans}$$

13. Given,

$$\lim_{x \rightarrow 0} \left(\frac{\sin 8x}{\tan 2x} \right), \quad \text{form} = \frac{0}{0} \\ = \lim_{x \rightarrow 0} \left(\frac{\sin 8x}{8x} \times \frac{2x}{\tan 2x} \times \frac{8}{2} \right) \\ = \left(1 \times 1 \times \frac{8}{2} \right) = 4 \quad \text{Ans}$$

14. Given,

$$\lim_{x \rightarrow 0} \frac{x + 1}{x - 1} \\ = \frac{0 + 1}{0 - 1} = -1 \quad \text{Ans}$$

15. Given,

$$\lim_{x \rightarrow 4} \frac{4x + 3}{x - 2} \\ = \frac{4(4) + 3}{4 - 2} = \frac{19}{2} \quad \text{Ans}$$

16. Given,

$$\lim_{x \rightarrow 1} \frac{x^{10} - 1}{x - 1}, \quad \text{form} = \frac{0}{0} \\ = \lim_{x \rightarrow 1} \frac{x^{10} - 1^{10}}{x - 1} \\ = 10 \times 1^9 \\ = 10 \times 1 = 10 \quad \text{Ans}$$

17. Given,

$$\lim_{x \rightarrow 0} \frac{x^5 + 2x - 8}{x - 8} \\ = \frac{0^5 + 2 \cdot (0) - 8}{0 - 8} \\ = \frac{-8}{-8} = 1 \quad \text{Ans}$$

18. Given,

$$\lim_{x \rightarrow 0} \frac{ax + b}{cx + 1} \\ = \frac{a \cdot (0) + b}{c \cdot (0) + 1} = b \quad \text{Ans}$$

19. Given,

$$\lim_{x \rightarrow 0} \frac{2x + 1}{5} \\ = \frac{2 \cdot (0) + 1}{5} = \frac{1}{5} \quad \text{Ans}$$

20. Given,

$$\lim_{x \rightarrow 0} \frac{5^x - 1}{x}, \quad \text{form} = \frac{0}{0} \\ = \log 5 \quad \text{Ans}$$

Short Answer Type Questions (लघु उत्तरीय प्रश्न)

Evaluate (मान ज्ञात करें)

1. $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right)$

2. $\lim_{x \rightarrow 0} \left(\frac{1 - \cos 4x}{1 - \cos 5x} \right)$

3. $\lim_{x \rightarrow 0} \left(\frac{ax + x \cos x}{b \sin x} \right)$

4. $\lim_{x \rightarrow 0} \left(\frac{x \cdot \tan x}{1 - \cos x} \right)$

5. $\lim_{x \rightarrow \frac{\pi}{2}} (\sec x - \tan x)$

6. $\lim_{x \rightarrow 0} \left(\frac{\sin ax + bx}{ax + \sin bx} \right)$

7. Find the value of $\lim_{x \rightarrow 0} f(x)$, where $f(x) = \frac{\sin 5x}{10x}$

$\lim_{x \rightarrow 0} f(x)$ का मान ज्ञात करें, जहाँ $f(x) = \frac{\sin 5x}{10x}$

8. Find the value of $\lim_{x \rightarrow 0} f(x)$, where $f(x) = \begin{cases} 2x + 3, & x \leq 0 \\ 3(x + 1), & x > 0 \end{cases}$

$\lim_{x \rightarrow 0} f(x)$ का मान ज्ञात करें, जहाँ $f(x) = \begin{cases} 2x + 3, & x \leq 0 \\ 3(x + 1), & x > 0 \end{cases}$

9. $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\cos x}{\frac{\pi}{2} - x} \right)$

10. $\lim_{x \rightarrow \pi} \left(\frac{1 + \cos x}{\tan^2 x} \right)$

Short Answer Type Questions (लघु उत्तरीय प्रश्न)

1. Given,

$$\begin{aligned} & \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right), \quad \text{form} = \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{x^2} \\ &= \lim_{x \rightarrow 0} 2 \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \times \frac{1}{2} \right)^2 \\ &= 2 \left(1 \times \frac{1}{2} \right)^2 = 2 \cdot \frac{1}{4} = \frac{1}{2} \quad \text{Ans} \end{aligned}$$

2. Given,

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{1 - \cos 5x}, \quad \text{form} = \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \frac{2 \sin^2 2x}{2 \sin^2 \frac{5x}{2}} \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \times \frac{5x/2}{\sin \frac{5x}{2}} \times 2 \times \frac{2}{5} \right)^2 \\ &= \left(1 \times 1 \times 2 \times \frac{2}{5} \right)^2 = \frac{16}{25} \quad \text{Ans} \end{aligned}$$

3. Given,

$$\begin{aligned} & \lim_{x \rightarrow 0} \left(\frac{ax + x \cdot \cos x}{b \sin x} \right), \quad \text{form} = \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \left(\frac{a + \cos x}{b} \right) \times \frac{x}{\sin x} \\ &= \left(\frac{a + \cos 0}{b} \right) \times (1) \\ &= \frac{a + 1}{b} \quad \text{Ans} \end{aligned}$$

4. Given,

$$\begin{aligned} & \lim_{x \rightarrow 0} \left(\frac{x \cdot \tan x}{1 - \cos x} \right), \quad \text{form} = \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \frac{x \tan x}{2 \sin^2 \frac{x}{2}} \\ &= \frac{1}{2} \lim_{x \rightarrow 0} x \times \frac{\tan x}{x} \times x \times \left(\frac{\frac{x}{2}}{\sin \frac{x}{2}} \times \frac{2}{x} \right)^2 \\ &= \frac{1}{2} \lim_{x \rightarrow 0} x^2 \left(\frac{\tan x}{x} \right) \cdot \left(\frac{\frac{x}{2}}{\sin \frac{x}{2}} \right)^2 \cdot \frac{4}{x^2} \\ &= \frac{1}{2} (1) \cdot (1)^2 \cdot 4 = 2 \quad \text{Ans} \end{aligned}$$

5. Given,

$$\begin{aligned} & \lim_{x \rightarrow \frac{\pi}{2}} (\sec x - \tan x) \\ &= \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x} \right) \\ &= \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{1 - \sin x}{\cos x} \right) \\ &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} \times \frac{1 + \sin x}{1 + \sin x} \\ &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin^2 x}{\cos x (1 + \sin x)} \\ &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x}{\cos x (1 + \sin x)} \\ &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{1 + \sin x} \\ &= \frac{\cos \frac{\pi}{2}}{1 + \sin \frac{\pi}{2}} = \frac{0}{1 + 1} = 0 \quad \text{Ans} \end{aligned}$$

6. Given,

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx}, \quad \text{form} &= \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \left(\frac{\frac{\sin ax}{ax} \times ax + bx}{ax + \frac{\sin bx}{bx} \times bx} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\frac{\sin ax}{ax} \times a + b}{a + \frac{\sin bx}{bx} \times b} \right) \\ &= \frac{1 \times a + b}{a + 1 \times b} = \frac{a + b}{a + b} = 1 \quad \text{Ans} \end{aligned}$$

7. Given,

$$\begin{aligned} f(x) &= \frac{\sin 5x}{10x} \\ \text{Now } \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{\sin 5x}{10x}, \quad \text{form} = \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} \times \frac{1}{2} \\ &= 1 \times \frac{1}{2} = \frac{1}{2} \quad \text{Ans} \end{aligned}$$

8. Given,

$$\begin{aligned} f(x) &= \begin{cases} 2x + 3, & x \leq 0 \\ 3(x + 1), & x > 0 \end{cases} \\ \text{Now, } \lim_{x \rightarrow 0^+} f(x) &= \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} 3 \cdot (h + 1) = 3 \\ \text{and, } \lim_{x \rightarrow 0^-} f(x) &= \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} [2(-h) + 3] = 3 \\ \text{clearly, } \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^-} f(x) = 3 \\ \text{Hence, } \lim_{x \rightarrow 0} f(x) &= 3 \quad \text{Ans} \end{aligned}$$

9. Given,

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\cos x}{\frac{\pi}{2} - x} \right), \quad \text{form} &= \frac{0}{0} \\ \text{let } \frac{\pi}{2} - x &= h \\ \text{as } x \rightarrow \frac{\pi}{2} \quad \therefore h &\rightarrow 0 \\ &= \lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} - h\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \quad \text{Ans} \end{aligned}$$

10. Given,

$$\lim_{x \rightarrow \pi} \left(\frac{1 + \cos x}{\tan^2 x} \right), \quad \text{form} = \frac{0}{0}$$

$$\begin{aligned} &= \lim_{x \rightarrow \pi} \frac{\cos^2 x (1 + \cos x)}{\sin^2 x} \\ &= \lim_{x \rightarrow \pi} \frac{\cos^2 x \cdot (1 + \cos x)}{1 - \cos^2 x} \\ &= \lim_{x \rightarrow \pi} \frac{\cos^2 x \cdot (1 + \cos x)}{(1 - \cos x)(1 + \cos x)} \\ &= \lim_{x \rightarrow \pi} \frac{\cos^2 x}{1 - \cos x} \\ &= \frac{\cos^2 \pi}{1 - \cos \pi} = \frac{(-1)^2}{1 - (-1)} = \frac{1}{1 + 1} = \frac{1}{2} \quad \text{Ans} \end{aligned}$$

Long Answer Type Questions (दीर्घ उत्तरीय प्रश्न)

Evaluate (मान ज्ञात करें)

- $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\tan 2x}{x - \frac{\pi}{2}} \right)$
- $\lim_{x \rightarrow 0} \left(\frac{\sin 2x + \sin 6x}{\sin 5x - \sin 3x} \right)$
- Let $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Find $\lim_{x \rightarrow 0} f(x)$

$$\text{माना } f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$\lim_{x \rightarrow 0} f(x)$ का मान ज्ञात करें।

- Let $f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$

If $\lim_{x \rightarrow 1} f(x) = f(1)$, find the value of a and b.

$$\text{माना } f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$$

यदि $\lim_{x \rightarrow 1} f(x) = f(1)$ है तो a तथा b ज्ञात करें।

- Show that $\lim_{x \rightarrow 0} \frac{x}{|x|}$ does not exist.

सिद्ध करें कि $\lim_{x \rightarrow 0} \frac{x}{|x|}$ अस्तित्व नहीं है।

Long Answer Type Questions
(दीर्घ उत्तरीय प्रश्न)

1. Given,

$$\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\tan 2x}{x - \frac{\pi}{2}} \right), \quad \text{form} = \frac{0}{0}$$

$$\text{Let } x - \frac{\pi}{2} = h$$

$$\text{as } x \rightarrow \frac{\pi}{2}$$

$$\therefore h \rightarrow 0$$

$$\begin{aligned} \text{Now, } \lim_{h \rightarrow 0} \frac{\tan 2\left(\frac{\pi}{2} + h\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\tan(\pi + 2h)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\tan 2h}{h} \\ &= \lim_{h \rightarrow 0} \left(\frac{\tan 2h}{2h} \right) \times 2 \\ &= (1) \times 2 \\ &= 2 \quad \text{Ans} \end{aligned}$$

2. Given,

$$\begin{aligned} \lim_{x \rightarrow 0} \left(\frac{\sin 2x + \sin 6x}{\sin 5x - \sin 3x} \right), \quad \text{form} = \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \left(\frac{2 \sin 4x \cdot \cos 2x}{2 \cos 4x \cdot \sin x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x \cdot \cos 2x}{\cos 4x \cdot \sin x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sin x} \times \frac{\cos 2x}{\cos 4x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \times 4x \times \frac{x}{\sin x} \times \frac{1}{x} \times \frac{\cos 2x}{\cos 4x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \times \frac{x}{\sin x} \times 4 \times \frac{\cos 2x}{\cos 4x} \right) \\ &= 1 \times 1 \times 4 \times \frac{1}{1} \\ &= 4 \quad \text{Ans} \end{aligned}$$

3. Given,

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

We have,

$$\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= \lim_{h \rightarrow 0} f(0 + h) \\ &= \lim_{h \rightarrow 0} \frac{|h|}{h} \end{aligned}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{h}{h} \\ &= 1 \end{aligned}$$

$$\text{and } \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h)$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{|-h|}{-h} \\ &= \lim_{h \rightarrow 0} \frac{h}{-h} \\ &= -1 \end{aligned}$$

clearly, (स्पष्टतः)

$$\lim_{x \rightarrow 0^+} f(x) \neq \lim_{x \rightarrow 0^-} f(x)$$

so, $\lim_{x \rightarrow 0} f(x)$ does not exist.

अतः $\lim_{x \rightarrow 0} f(x)$ अस्तित्व नहीं है।

4. Given,

$$f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$$

$$f(1) = 4$$

$$\begin{aligned} \lim_{x \rightarrow 1^+} f(x) &= \lim_{h \rightarrow 0} f(1 + h) \\ &= \lim_{h \rightarrow 0} [b - a(1 + h)] \\ &= b - a \end{aligned}$$

$$\begin{aligned} \text{and } \lim_{x \rightarrow 1^-} f(x) &= \lim_{h \rightarrow 0} f(1 - h) \\ &= \lim_{h \rightarrow 0} [a + b(1 - h)] \\ &= a + b \end{aligned}$$

$$\text{Now, } \lim_{x \rightarrow 1} f(x) = f(1) = 4$$

$$\begin{aligned} \lim_{x \rightarrow 1^+} f(x) &= \lim_{x \rightarrow 1^-} f(x) = f(1) \\ \Rightarrow b - a &= a + b = 4 \\ \Rightarrow b - a &= 4 \quad \text{and} \quad a + b = 4 \\ \Rightarrow a &= 0, b = 4 \\ \text{so, } a &= 0 \text{ and } b = 4 \quad \text{Ans} \end{aligned}$$

5. Given,

$$\text{Let } f(x) = \frac{x}{|x|}. \quad \text{Then,}$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h)$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{h}{|h|} \\ &= \lim_{h \rightarrow 0} \frac{h}{h} \\ &= 1 \end{aligned}$$

$$\begin{aligned}
\text{and } \lim_{x \rightarrow 0^-} f(x) &= \lim_{h \rightarrow 0} f(0 - h) \\
&= \lim_{h \rightarrow 0} \frac{-h}{|-h|} \\
&= \lim_{h \rightarrow 0} \frac{-h}{h} \\
&= -1
\end{aligned}$$

clearly, (स्पष्टतः)

$$\lim_{x \rightarrow 0^+} f(x) \neq \lim_{x \rightarrow 0^-} f(x)$$

Hence, $\lim_{x \rightarrow 0} f(x)$ does not exist.

अतः $\lim_{x \rightarrow 0} f(x)$ अस्तित्व नहीं है।

DIFFERENTIATION

अवकलन

Derivatives (अवकलज)

1. Differentiate with respect to x (d.w.r. to x) = $\frac{d}{dx}$
 x के सापेक्ष अवकलज = $\frac{d}{dx}$

2. Differentiation from the first principle

प्रथम सिद्धांत से अवकलन

$$y = f(x)$$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

3. Algebra of derivative of funtions

फलनों के अवकज का बीजगणित

If u and v are funtions of x then

यदी u तथा v x के फलन हैं तो

i. $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

ii. $\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$

iii. $\frac{d}{dx}(u \cdot v) = u \frac{dv}{dx} + v \frac{du}{dx}$

iv. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

v. $\frac{d}{dx}cu = c \cdot \frac{du}{dx}, \quad c = \text{constant. (नियत)}$

4. Some important Derivatives Furmula

अवकलज के कुछ महत्वपूर्ण सूत्र

i. $\frac{d}{dx}c = 0, \quad c = \text{constant.}$

ii. $\frac{d}{dx}(x^n) = nx^{n-1}$

iii. $\frac{d}{dx}(x) = 1$

iv. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

v. $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

vi. $\frac{d}{dx}(a^x) = a^x \cdot \log a$

vii. $\frac{d}{dx}(e^x) = e^x$

viii. $\frac{d}{dx}(\log x) = \frac{1}{x}$

ix. $\frac{d}{dx}(\sin x) = \cos x$

x. $\frac{d}{dx}(\cos x) = -\sin x$

xi. $\frac{d}{dx}(\tan x) = \sec^2 x$

xii. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

xiii. $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

xiv. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$

Multiple Choice Questions

(बहु विकल्पीय प्रश्न)

1. $\frac{d}{dx}(x^{10}) =$

- (a) $10x^9$ (b) x^9
 (C) $10x^8$ (d) x^{10}

2. $\frac{d}{dx}(2\sqrt{x}) =$

- (a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{1}{\sqrt{x}}$
 (c) $\frac{2}{\sqrt{x}}$ (d) $\sqrt{x}$

3. $\frac{d}{dx}(5e^x) =$

- (a) e^x (b) $\frac{5}{e^x}$
 (c) $5e^x$ (d) None of these

4. $\frac{d}{dx}(5x) =$

- (a) 5 (b) $5x$
 (c) x (d) 0

5. $\frac{d}{dx}(8) =$

- (a) 8 (b) 1
 (c) -8 (d) 0

6. $\frac{d}{dx}(2^x) =$

- (a) 2^x (b) $2^x \cdot \log 2$
 (c) $\log 2$ (d) 0

7. $\frac{d}{dx}\left(\frac{1}{x}\right) =$

- (a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$
 (c) $\frac{-1}{x^2}$ (d) x^2

8. $\frac{d}{dx}(x^{-2}) =$

- (a) x^{-3} (b) $-2x^{-3}$
 (c) $-2x^{-1}$ (d) $2x$

9. $\frac{d}{dx}(2\log x) =$

- (a) $\frac{2}{x}$ (b) $\frac{1}{x}$
 (c) $2x$ (d) x

10. $\frac{d}{dx}(x^{3/2}) =$

- (a) $\frac{3}{2}x^{1/2}$ (b) $x^{1/2}$
 (c) $\frac{3}{2}x^{5/2}$ (d) $\frac{3}{2}x$
11. $\frac{d}{dx}(2x+8) =$
 (a) 0 (b) 1
 (c) 2 (d) 8
12. $\frac{d}{dx}(x^2+4x+1) =$
 (a) $2x+1$ (b) $2x+4$
 (c) $2x$ (d) $4x+2$
13. $\frac{d}{dx}(3x^2-8) =$
 (a) $6x^2$ (b) 6
 (c) $6x$ (d) $3x$
14. $\frac{d}{dx}(e^x+1) =$
 (a) e^x (b) e^{-x}
 (c) $x \cdot e^x$ (d) $x \cdot e^{x-1}$
15. If $y=5x^2-2x+11$ then $\frac{dy}{dx} =$
 यदि $y=5x^2-2x+11$ तो $\frac{dy}{dx} =$
 (a) $5x-2$ (b) $10x-2$
 (c) $10x$ (d) $10x+2$
16. If $y=\sin x+\cos x$ then $\frac{dy}{dx} =$
 यदि $y=\sin x+\cos x$ तो $\frac{dy}{dx} =$
 (a) $\cos x-\sin x$ (b) $\sin x+\cos x$
 (c) $2\sin x$ (d) $2\cos x$
17. If $y=e^x-x$ then $\frac{dy}{dx} =$
 (a) e^x (b) e^x+1
 (c) e^x-1 (d) -1
18. If $y=x^2-2$ then $\frac{dy}{dx} =$
 (a) $2x$ (b) $2x-1$
 (c) 2 (d) 0
19. If $y=99x$ then $\frac{dy}{dx} =$
 (a) 100 (b) 99
 (c) $99x$ (d) 0
20. If $y=x^{2023}$ then $\frac{dy}{dx} =$
 (a) $2023x^{2023}$ (b) $2023x^{2022}$
 (c) $2023x^{2021}$ (d) $2023x^{2024}$
21. If $y=x^{-7}+8$ then $\frac{dy}{dx} =$
 (a) x^{-7} (b) $-7x^{-6}$
 (c) $-7x^{-8}$ (d) x^{-8}
22. If $y = x^{\frac{5}{2}} + x^{\frac{3}{2}} + x$ then $\frac{dy}{dx} =$
 (a) $\frac{5}{2}x^{3/2} + \frac{3}{2}\sqrt{x} + 1$
 (b) $x^{3/2} + \sqrt{x} + 1$
 (c) $\frac{5}{2}x^{3/2} + \sqrt{x} + 1$
 (d) None of these.
23. $\frac{d}{dx}e^{4x} =$
 (a) e^{4x} (b) $4 \cdot e^{4x}$
 (c) $\frac{e^{4x}}{4}$ (d) e^{-4x}
24. $\frac{d}{dx}\sin 5x =$
 (a) $\cos 5x$ (b) $-\cos 5x$
 (c) $5\cos 5x$ (d) $5\sin 5x$
25. $\frac{d}{dx}\sqrt{5x} =$
 (a) $\frac{1}{2\sqrt{5x}}$ (b) $2\sqrt{5x}$
 (c) $\frac{2}{5\sqrt{x}}$ (d) $\frac{5}{2\sqrt{5x}}$
26. $\frac{d}{dx}\tan 2x =$
 (a) $2\cot 2x$ (b) $2\sec^2 2x$
 (c) $2\tan 2x$ (d) $\sec^2 2x$
27. $\frac{d}{dx}e^{-x} =$
 (a) $-e^{-x}$ (b) e^{-x}
 (c) e^x (d) $-e^x$
28. $\frac{d}{dx}\log 5x =$
 (a) $\frac{1}{5x}$ (b) $\frac{1}{x}$
 (c) $\frac{5}{x}$ (d) None of these.
29. $\frac{d}{dx}\log(\sin x) =$
 (a) $\tan x$ (b) $-\tan x$
 (c) $\cot x$ (d) $-\cot x$
30. $\frac{d}{dx}\log(\cos x) =$
 (a) $\tan x$ (b) $-\tan x$
 (c) $\cot x$ (d) $-\cot x$
31. $\frac{d}{dx}\sqrt{\sin x} =$
 (a) $\frac{1}{2\sqrt{\sin x}}$ (b) $2\sqrt{\sin x}$
 (c) $\frac{\sin x}{2\sqrt{\cos x}}$ (d) $\frac{\cos x}{2\sqrt{\sin x}}$

32. $\frac{d}{dx}\sqrt{x+2} =$
 (a) $2\sqrt{x+2}$ (b) $\frac{1}{2\sqrt{x+2}}$
 (c) $\frac{1}{\sqrt{x+2}}$ (d) None of these.
33. $\frac{d}{dx}\log(2x+1) =$
 (a) $\frac{2}{2x+1}$ (b) $\frac{1}{2x+1}$
 (c) $2(2x+1)$ (d) $\frac{-1}{2x+1}$
34. $\frac{d}{dx}x \cdot e^x =$
 (a) $(x+1)$ (b) $e^x \cdot (x+1)$
 (c) e^{x+1} (d) e^x
35. $\frac{d}{dx}x \cdot \sin x =$
 (a) $x\sin x + \cos x$ (b) $x\cos x - \sin x$
 (c) $x\cos x + \sin x$ (d) $x\sin x - \cos x$
36. $\frac{d}{dx}x \cdot \log x$
 (a) $1 + \log x$ (b) $\log x$
 (c) $1 - \log x$ (d) $\log x - 1$
37. The derivative of the function $\sin(x^2+1)$ w.r. to x is
 फलन $\sin(x^2+1)$ का अवकलन गुणांक के सापेक्ष होगा
 (a) $\cos(x^2+1)$ (b) $2x\cos(x^2+1)$
 (c) $\cos(2x^3+2x)$ (d) $\sin(x^2+1)$
38. The derivative of the function $\cos x^3$ w.r. to x is
 फलन $\cos x^3$ का अवकलन गुणांक x के सापेक्ष होगा
 (a) $3x^2\sin x^3$ (b) $-3x^2\sin x^3$
 (c) $x^2\sin x^3$ (d) None of these.
39. The value of $\frac{d}{dx}e^{x^4}$ will be
 $\frac{d}{dx}e^{x^4}$ का मान होगा
 (a) $4x^3 \cdot e^{x^4}$ (b) $x^3 \cdot e^{x^4}$
 (c) $4x^3 \cdot e^{x^3}$ (d) e^{x^4}
40. The value of $\frac{d}{dx}(3x^2 - 6x)$ will be
 $\frac{d}{dx}(3x^2 - 6x)$ का मान होगा
 (a) $3x-6$ (b) $6x-6$
 (c) $6x$ (d) $x-1$
41. $\frac{d}{dx}(x^2 \cdot \sin x) =$
 (a) $2x\sin x$ (b) $2x\sin x + x^2\cos x$
 (c) $2x\cos x + x^2\sin x$ (d) $2x\cos x$
42. $\frac{d}{dx}\sin^2 x =$
 (a) $\sin 2x$ (b) $\sin x$
 (c) $2\sin x$ (d) $\cos 2x$
43. $\frac{d}{dx}\sin x \cdot \cos x =$
 (a) $\cos x$ (b) $\sin x$
 (c) $\cos 2x$ (d) $\sin 2x$
44. $\frac{d}{dx}\left(\frac{x}{x+1}\right) =$
 (a) $\frac{1}{(x+1)^2}$ (b) $\frac{x}{(x+1)^2}$
 (c) $\frac{-1}{(x+1)^2}$ (d) $\frac{-x}{(x+1)^2}$
45. If $f(x) = 2x^2 + 3x - 5$ then $f'(0) + 3 \cdot f'(-1) =$
 यदि $f(x) = 2x^2 + 3x - 5$ तो $f'(0) + 3 \cdot f'(-1) =$
 (a) 2 (b) 0
 (c) 1 (d) -1
46. If $f(x) = \sin x$ then $f'\left(\frac{\pi}{2}\right) =$
 यदि $f(x) = \sin x$ तो $f'\left(\frac{\pi}{2}\right) =$
 (a) 2 (b) 0
 (c) 1 (d) -1
47. If $f(x) = 2x^2 - 2x + 1$ then $f'(0) + f'(2) =$
 $f(x) = 2x^2 - 2x + 1$ तो $f'(0) + f'(2) =$
 (a) 2 (b) 4
 (c) -2 (d) 0
48. $\frac{d}{dx}\left(\frac{\sin x}{x}\right) =$
 (a) $\frac{\sin x - \cos x}{x^2}$ (b) $\frac{\sin x - x\cos x}{x^2}$
 (c) $\frac{x\sin x - \cos x}{x^2}$ (d) None of these.
49. $\frac{d}{dx}\sin\sqrt{x} =$
 (a) $\frac{\sin\sqrt{x}}{2\sqrt{x}}$ (b) $\frac{\cos\sqrt{x}}{\sqrt{x}}$
 (c) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$ (d) $\cos\sqrt{x}$
50. $\frac{d}{dx}\tan x^2 =$
 (a) $\sec x^2$ (b) $2x\tan x^2$
 (c) $2x\sec x$ (d) $2x\sec^2 x^2$

51. If $y = \cos \sqrt{x}$ then value of $\frac{dy}{dx}$ will be

यदि $y = \cos \sqrt{x}$ तो $\frac{dy}{dx}$ का मान होगा

- (a) $\frac{-\sin \sqrt{x}}{2\sqrt{x}}$ (b) $\frac{\sin \sqrt{x}}{2\sqrt{x}}$
 (c) $\frac{\cos \sqrt{x}}{2\sqrt{x}}$ (d) None of these.

52. If $f(x) = 4x^3 - 2x + 8$ then $f'(0) =$
 यदि $f(x) = 4x^3 - 2x + 8$ तो $f'(0) =$

- (a) 10 (b) 12
 (c) -2 (d) 8

53. If $f(x) = e^x \sin x$ then $f'(x)$ will be
 यदि $f(x) = e^x \sin x$ तो $f'(x)$ का मान होगा

- (a) $e^x(\sin x + \cos x)$ (b) $e^x(\sin x - \cos x)$
 (c) $e^x(\cos x - \sin x)$ (d) None of these.

54. $\frac{d}{dx} x^2 \cdot e^x =$

- (a) $e^x(x+2)$ (b) $e^x(x^2+2x)$
 (c) $e^x(x^2+1)$ (d) $e^x(x^2-2x)$

55. $\frac{d}{dx} e^{\cos x} =$

- (a) $-e^{\cos x}$ (b) $-\sin x \cdot e^{\cos x}$
 (c) $\cos x \cdot e^{\cos x}$ (d) None of these.

56. $\frac{d}{dx} (5 + 7x)^6 =$

- (a) $6(5+7x)^5$ (b) $7(5+7x)^6$
 (c) $42(5+7x)^5$ (d) $42(5+7x)^7$

57. $\frac{d}{dx} \sec x =$

- (a) $\operatorname{cosec} x$ (b) $\sec x \cdot \cot x$
 (c) $\sec x \cdot \tan x$ (d) None of these.

58. If $f(x) = \tan x$ then $f'(0) =$
 यदि $f(x) = \tan x$ तो $f'(0) =$

- (a) 0 (b) -1
 (c) 2 (d) 1

59. $\frac{d}{dx} x^{-5/2} =$

- (a) $\frac{5}{2} x^{-7/2}$ (b) $-\frac{5}{2} x^{-7/2}$
 (c) $-\frac{5}{2} x^{-3/2}$ (d) None of these.

60. $\frac{d}{dx} \cos^2 x =$

- (a) $-\sin 2x$ (b) $\sin 2x$
 (c) $2 \sin 2x$ (d) $\cos 2x$

Answer key उत्तरमाला

Multiple Choice Questions (बहु विकल्पीय प्रश्न)

- | | | | | |
|--------|--------|--------|--------|--------|
| (1) a | (2) b | (3) c | (4) a | (5) d |
| (6) b | (7) c | (8) b | (9) a | (10) a |
| (11) c | (12) b | (13) c | (14) a | (15) b |
| (16) a | (17) c | (18) a | (19) b | (20) b |
| (21) c | (22) a | (23) b | (24) c | (25) d |
| (26) b | (27) a | (28) b | (29) c | (30) b |
| (31) d | (32) b | (33) a | (34) b | (35) c |
| (36) a | (37) b | (38) b | (39) a | (40) b |
| (41) b | (42) a | (43) c | (44) a | (45) b |
| (46) b | (47) b | (48) d | (49) c | (50) d |
| (51) a | (52) c | (53) a | (54) b | (55) b |
| (56) c | (57) c | (58) d | (59) b | (60) a |

Very Short Answer Type Questions (अति लघु उत्तरीय प्रश्न)

Find $\frac{dy}{dx}$ ($\frac{dy}{dx}$ ज्ञात करें)

- | | |
|--------------------------|---|
| 1. $y = x^{-3}$ | 16. $y = \frac{1}{x^2}$ |
| 2. $y = x^{1/3}$ | 17. $y = \sec x - \tan x$ |
| 3. $y = 2x^2 + 4x + 1$ | 18. $y = 5 \sin x + 6 \tan x$ |
| 4. $y = 10x$ | 19. $y = 6x^2 - 12x - 2023$ |
| 5. $y = x^3 - 27$ | 20. $y = 5x - 9x^2$ |
| 6. $y = (x-1)(x-2)$ | 21. $y = x + x^2 + x^3$ |
| 7. $y = \tan x + \cot x$ | 22. $y = e^{3x}$ |
| 8. $y = \sin x + e^x$ | 23. $y = 2x$ |
| 9. $y = (5x+1)^2$ | 24. $y = e^{-x}$ |
| 10. $y = \sin 3x$ | 25. $y = x \cdot \tan x$ |
| 11. $y = \sqrt{3x}$ | 26. $y = x^{-3} + 2x + 1$ |
| 12. $y = \sqrt{\tan x}$ | 27. $y = \operatorname{cosec} x - \sqrt{x}$ |
| 13. $y = \cos 9x$ | 28. $y = \log x - 2e^x$ |
| 14. $y = x^3 - 4x + 8$ | 29. $y = x^{3/2} - 2\sqrt{x} + 7$ |
| 15. $y = \frac{1}{x}$ | 30. $y = \log 4x$ |

Very Short Answer Type Questions (अति लघु उत्तरीय प्रश्न)

1. Given, $y = x^{-3}$
 d.w.r. to x

1. $\frac{dy}{dx} = -3x^{-4}$ Ans
2. Given, $y = x^{1/3}$
d.w.r. to x
 $\frac{dy}{dx} = \frac{1}{3}x^{-2/3}$ Ans
3. Given, $y = 2x^2 + 4x + 1$
d.w.r. to x
 $\frac{dy}{dx} = \frac{d}{dx}(2x^2 + 4x + 1)$
 $= 2 \cdot \frac{dx^2}{dx} + 4 \cdot \frac{dx}{dx} + \frac{d(1)}{dx}$
 $= 2 \cdot (2x) + 4 \cdot (1) + (0)$
 $= 4x + 4$ Ans
4. Given, $y = 10x$
d.w.r. to x
 $\frac{dy}{dx} = \frac{d}{dx} 10x$
 $= 10 \cdot \frac{dx}{dx}$
 $= 10 \cdot (1)$
 $= 10$ Ans
5. Given, $y = x^3 - 27$
d.w.r. to x
 $\frac{dy}{dx} = 3x^2 - 0 = 3x^2$ Ans
6. Given, $y = (x-1) \cdot (x-2)$
 $y = x^2 - 2x - x + 2$
 $y = x^2 - 3x + 2$
d.w.r. to x
 $\frac{dy}{dx} = \frac{d}{dx}(x^2 - 3x + 2)$
 $= 2x - 3 + 0$
 $= 2x - 3$ Ans
7. Given, $y = \tan x + \cot x$
d.w.r. to x
 $\frac{dy}{dx} = \frac{d}{dx} \tan x + \frac{d}{dx} \cot x$
 $= \sec^2 x - \operatorname{cosec}^2 x$ Ans
8. Given, $y = \sin x + e^x$
d.w.r. to x
 $\frac{dy}{dx} = \frac{d}{dx}(\sin x + e^x)$
 $= \frac{d}{dx} \sin x + \frac{d}{dx} e^x$
 $= \cos x + e^x$ Ans

9. Given, $y = (5x+1)^2$
d.w.r. to x
 $\frac{dy}{dx} = \frac{d}{dx}(5x+1)^2$
 $= \frac{d}{d(5x+1)}(5x+1)^2 \times \frac{d}{dx}(5x+1)$
 $= 2 \cdot (5x+1) \times (5)$
 $= 10(5x+1)$ Ans
10. Given, $y = \sin 3x$
d.w.r. to x
 $\frac{dy}{dx} = \frac{d}{dx} \sin 3x$
 $= \frac{d}{d(3x)}(\sin 3x) \times \frac{d}{dx}(3x)$
 $= \cos 3x \times 3 = 3 \cos 3x$ Ans
11. Given, $y = \sqrt{3x}$
d.w.r. to x
 $\frac{dy}{dx} = \frac{d}{dx} \sqrt{3x}$
 $= \frac{d}{d(3x)}(\sqrt{3x}) \times \frac{d}{dx} 3x$
 $= \frac{1}{2\sqrt{3x}} \times 3$
 $= \frac{3}{2\sqrt{3x}}$ Ans
12. Given, $y = \sqrt{\tan x}$
d.w.r. to x
 $\frac{dy}{dx} = \frac{d}{dx} \sqrt{\tan x}$
 $= \frac{d}{d(\tan x)}(\sqrt{\tan x}) \times \frac{d}{dx} \tan x$
 $= \frac{1}{2\sqrt{\tan x}} \times \sec^2 x$
 $= \frac{\sec^2 x}{2\sqrt{\tan x}}$ Ans
13. Given, $y = \cos 9x$
d.w.r. to x
 $\frac{dy}{dx} = \frac{d}{dx} \cos 9x$
 $= \frac{d}{d(9x)} \cos 9x \times \frac{d}{dx} 9x$
 $= (-\sin 9x) \times (9)$
 $= -9 \sin 9x$ Ans

14. Given, $y=x^3-4x+8$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 4x + 8)$$

$$= 3x^2 - 4 \quad \text{Ans}$$

15. Given, $y=\frac{1}{x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{-1}{x^2} \quad \text{Ans}$$

16. Given, $y=\frac{1}{x^2}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}x^{-2}$$

$$= -2x^{-3} \quad \text{Ans}$$

17. Given, $y=\sec x - \tan x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx} \sec x - \frac{d}{dx} \tan x$$

$$= \sec x \cdot \tan x - \sec^2 x$$

$$\sec x \cdot (\tan x - \sec x) \quad \text{Ans}$$

18. Given, $y=5\sin x+6\tan x$

d.w.r. to x

$$\frac{dy}{dx} = 5 \cdot \frac{d\sin x}{dx} + 6 \cdot \frac{d\tan x}{dx}$$

$$= 5\cos x + 6\sec^2 x \quad \text{Ans}$$

19. Given, $y=6x^2-12x-2023$

d.w.r. to x

$$\frac{dy}{dx} = 6 \cdot \frac{dx^2}{dx} - 12 \cdot \frac{dx}{dx} - \frac{d}{dx}(2023)$$

$$= 12x - 12 - 0$$

$$= 12(x - 1) \quad \text{Ans}$$

20. Given, $y=5x-9x^2$

d.w.r. to x

$$\frac{dy}{dx} = 5 \frac{dx}{dx} - 9 \frac{dx^2}{dx}$$

$$= 5 \cdot (1) - 9(2x)$$

$$= 5 - 18x \quad \text{Ans}$$

21. Given, $y=x+x^2+x^3$

d.w.r. to x

$$\frac{dy}{dx} = 1 + 2x + 3x^2 \quad \text{Ans}$$

22. Given, $y = e^{3x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx} e^{3x}$$

$$= \frac{d}{d(3x)} e^{3x} \times \frac{d}{dx} 3x$$

$$= e^{3x} \cdot 3 = 3e^{3x} \quad \text{Ans}$$

23. Given, $y=2x$

d.w.r. to x

$$\frac{dy}{dx} = 2 \cdot \frac{dx}{dx} = 2(1) = 2 \quad \text{Ans}$$

24. Given, $y=e^{-x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx} e^{-x}$$

$$= \frac{d}{d(-x)} (e^{-x}) \times \frac{d}{dx} (-x)$$

$$= e^{-x} \times (-1) = -e^{-x} \quad \text{Ans}$$

25. Given, $y=x \cdot \tan x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx} (x \cdot \tan x)$$

$$= x \cdot \frac{d}{dx} \tan x + \tan x \frac{dx}{dx}$$

$$= x \sec^2 x + \tan x(1)$$

$$= x \sec^2 x + \tan x \quad \text{Ans}$$

26. Given, $y=x^{-3}+2x+1$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx} x^{-3} + 2 \cdot \frac{dx}{dx} + \frac{d}{dx} (1)$$

$$= -3x^{-4} + 2 \quad \text{Ans}$$

27. Given, $y=\operatorname{cosec} x - \sqrt{x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx} \operatorname{cosec} x - \frac{d}{dx} \sqrt{x}$$

$$= -\operatorname{cosec} x \cdot \cot x - \frac{1}{2\sqrt{x}} \quad \text{Ans}$$

28. Given, $y=\log x - 2e^x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx} \log x - 2 \cdot \frac{d}{dx} e^x$$

$$= \frac{1}{x} - 2e^x \quad \text{Ans}$$

29. Given, $y=x^{3/2} - 2\sqrt{x} + 7$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} x^{3/2} - 2 \frac{d}{dx} \sqrt{x} + \frac{d}{dx} (7) \\ &= \frac{3}{2} x^{1/2} - 2 \cdot \frac{1}{2\sqrt{x}} + 0 \\ &= \frac{3\sqrt{x}}{2} - \frac{1}{\sqrt{x}} \quad \text{Ans}\end{aligned}$$

30. Given, $y = \log 4x$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \log 4x \\ &= \frac{d}{d(4x)} \log 4x \times \frac{d}{dx} (4x) \\ &= \left(\frac{1}{4x} \right) \times 4 \cdot (1) \\ &= \frac{1}{x} \quad \text{Ans}\end{aligned}$$

Short Answer Type Questions (लघु उत्तरीय प्रश्न)

Find $\frac{dy}{dx}$ ($\frac{dy}{dx}$ ज्ञात करें)

- | | |
|---------------------------|------------------------------------|
| 1. $y = x \cdot \sec x$ | 16. $y = e^{4x-8}$ |
| 2. $y = x^2 \cdot \cos x$ | 17. $y = e^{8x+8}$ |
| 3. $y = x^3 \cdot e^x$ | 18. $y = \log(x^2+1)$ |
| 4. $y = x \cdot \log x$ | 19. $y = \cos(5x^2+1)$ |
| 5. $y = e^x \cdot \sin x$ | 20. $y = 3^x - e^{2x}$ |
| 6. $y = e^x \cdot \tan x$ | 21. $y = 5 \sec x + 4 \cos x$ |
| 7. $y = \frac{\sin x}{x}$ | 22. $y = x^{-4} \cdot (3-4x^{-5})$ |
| 8. $y = \frac{x+1}{x}$ | 23. $y = x^{-3} \cdot (5+3x)$ |
| 9. $y = \frac{x+1}{x-1}$ | 24. $y = x^5 \cdot (3-6x^{-9})$ |
| 10. $y = \sin^3 x$ | 25. $y = 2x - \frac{3}{4}$ |
| 11. $y = \sin x^2$ | 26. $y = 5 \sin x - 6 \cos x + 7$ |
| 12. $y = \sqrt{\sec x}$ | 27. $y = 2 \tan x - 7 \sec x$ |
| 13. $y = \sqrt{\cot x}$ | 28. $y = x + \frac{1}{x}$ |
| 14. $y = \sin(2x-5)$ | 29. $y = \frac{x^2 + 2x + 1}{x}$ |
| 15. $y = \log(7x+1)$ | 30. $y = 4\sqrt{x} - 2$ |

Short Answer Type Questions (लघु उत्तरीय प्रश्न)

1. Given, $y = x \cdot \sec x$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (x \cdot \sec x) \\ &= x \cdot \frac{d}{dx} \sec x + \sec x \cdot \frac{d}{dx} x \\ &= x \cdot (\sec x \cdot \tan x) + \sec x \cdot (1) \\ &= \sec x \cdot (x \cdot \tan x + 1) \quad \text{Ans}\end{aligned}$$

2. Given, $y = x^2 \cdot \cos x$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (x^2 \cdot \cos x) \\ &= x^2 \cdot \frac{d}{dx} \cos x + \cos x \cdot \frac{d}{dx} x^2 \\ &= x^2 \cdot (-\sin x) + \cos x \cdot (2x) \\ &= x \cdot (2 \cos x - x \sin x) \quad \text{Ans}\end{aligned}$$

3. Given, $y = x^3 \cdot e^x$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (x^3 \cdot e^x) \\ &= x^3 \cdot \frac{d}{dx} e^x + e^x \cdot \frac{d}{dx} x^3 \\ &= x^3 \cdot (e^x) + e^x \cdot (3x^2) \\ &= x^2 \cdot e^x (x + 3) \quad \text{Ans}\end{aligned}$$

4. Given, $y = x \cdot \log x$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= x \cdot \frac{d}{dx} \log x + \log x \cdot \frac{d}{dx} x \\ &= x \cdot \left(\frac{1}{x} \right) + \log x \cdot (1) \\ &= 1 + \log x \quad \text{Ans}\end{aligned}$$

5. Given, $y = e^x \cdot \sin x$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (e^x \cdot \sin x) \\ &= e^x \cdot \frac{d}{dx} \sin x + \sin x \cdot \frac{d}{dx} e^x \\ &= e^x \cdot (\cos x) + \sin x \cdot (e^x) \\ &= e^x \cdot (\cos x + \sin x) \quad \text{Ans}\end{aligned}$$

6. Given, $y = e^x \cdot \tan x$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(e^x \cdot \tan x) \\ &= e^x \cdot \frac{d}{dx} \tan x + \tan x \cdot \frac{d}{dx} e^x \\ &= e^x (\sec^2 x) + \tan x \cdot (e^x) \\ &= e^x (\sec^2 x + \tan x) \quad \text{Ans}\end{aligned}$$

7. Given, $y = \frac{\sin x}{x}$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{\sin x}{x} \right) \\ &= \frac{x \frac{d}{dx} \sin x - \sin x \cdot \frac{d}{dx} x}{x^2} \\ &= \frac{x \cdot (\cos x) - \sin x \cdot (1)}{x^2} \\ &= \frac{x \cdot \cos x - \sin x}{x^2} \quad \text{Ans}\end{aligned}$$

8. Given, $y = \frac{x+1}{x}$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{x+1}{x} \right) \\ &= \frac{x \cdot \frac{d}{dx} (x+1) - (x+1) \frac{d}{dx} x}{x^2} \\ &= \frac{x \cdot (1+0) - (x+1) \cdot (1)}{x^2} \\ &= \frac{x - x - 1}{x^2} \\ &= \frac{-1}{x^2} \quad \text{Ans}\end{aligned}$$

9. Given, $y = \frac{x+1}{x-1}$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{x+1}{x-1} \right) \\ &= \frac{(x-1) \frac{d}{dx} (x+1) - (x+1) \frac{d}{dx} (x-1)}{(x-1)^2} \\ &= \frac{(x-1)(1+0) - (x+1)(1-0)}{(x-1)^2} \\ &= \frac{x-1-x-1}{(x-1)^2} \\ &= \frac{-2}{(x-1)^2} \quad \text{Ans}\end{aligned}$$

10. Given, $y = \sin^3 x$

$$y = (\sin x)^3$$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (\sin x)^3 \\ &= \frac{d}{d(\sin x)} (\sin x)^3 \times \frac{d}{dx} (\sin x) \\ &= 3 \cdot (\sin x)^2 \times (\cos x) \\ &= 3 \sin^2 x \cdot \cos x \quad \text{Ans}\end{aligned}$$

11. Given, $y = \sin x^2$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \sin x^2 \\ &= \frac{d \sin x^2}{d(x^2)} \times \frac{d}{dx} (x^2) \\ &= (\cos x^2) \times (2x) \\ &= 2x \cdot \cos x^2 \quad \text{Ans}\end{aligned}$$

12. Given, $y = \sqrt{\sec x}$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \sqrt{\sec x} \\ &= \frac{d}{d(\sec x)} \sqrt{\sec x} \times \frac{d}{dx} (\sec x) \\ &= \frac{1}{2\sqrt{\sec x}} \times \sec x \cdot \tan x \\ &= \frac{\sqrt{\sec x} \cdot \tan x}{2} \quad \text{Ans}\end{aligned}$$

13. Given, $y = \sqrt{\cot x}$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \sqrt{\cot x} \\ &= \frac{d}{d(\cot x)} \sqrt{\cot x} \times \frac{d}{dx} \cot x \\ &= \frac{1}{2\sqrt{\cot x}} \times (-\operatorname{cosec}^2 x) \\ &= \frac{-\operatorname{cosec}^2 x}{2\sqrt{\cot x}} \quad \text{Ans}\end{aligned}$$

14. Given, $y = \sin(2x-5)$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \sin(2x-5) \\ &= \frac{d}{d(2x-5)} \sin(2x-5) \times \frac{d}{dx} (2x-5) \\ &= \cos(2x-5) \times (2-0) \\ &= 2\cos(2x-5) \quad \text{Ans}\end{aligned}$$

15. Given, $y = \log(7x+1)$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \log(7x+1) \\ &= \frac{d}{d(7x+1)} \log(7x+1) \times \frac{d}{dx}(7x+1) \\ &= \frac{1}{7x+1} \times (7+0) \\ &= \frac{7}{7x+1} \quad \text{Ans}\end{aligned}$$

16. Given, $y = e^{4x-8}$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} e^{4x-8} \\ &= \frac{d}{d(4x-8)} e^{4x-8} \times \frac{d}{dx}(4x-8) \\ &= e^{4x-8} \times (4-0) \\ &= 4 \cdot e^{4x-8} \quad \text{Ans}\end{aligned}$$

17. Given, $y = e^{8x+8}$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} e^{8x} + \frac{d}{dx}(8) \\ &= \frac{d}{d8x} e^{8x} \times \frac{d}{dx}(8x) + \frac{d}{dx}(8) \\ &= e^{8x} \times 8 + 0 \\ &= 8e^{8x} \quad \text{Ans}\end{aligned}$$

18. Given, $y = \log(x^2+1)$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \log(x^2+1) \\ &= \frac{d}{d(x^2+1)} \log(x^2+1) \times \frac{d}{dx}(x^2+1) \\ &= \frac{1}{x^2+1} \times (2x+0) \\ &= \frac{2x}{x^2+1} \quad \text{Ans}\end{aligned}$$

19. Given, $y = \cos(5x^2+1)$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \cos(5x^2+1) \\ &= \frac{d}{d(5x^2+1)} \cos(5x^2+1) \times \frac{d}{dx}(5x^2+1) \\ &= -\sin(5x^2+1) \times (10x+0) \\ &= -10x \cdot \sin(5x^2+1) \quad \text{Ans}\end{aligned}$$

20. Given, $y = 3^x - e^{2x}$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} 3^x - \frac{d}{dx} e^{2x} \\ &= 3^x \cdot \log 3 - 2e^{2x} \quad \text{Ans}\end{aligned}$$

21. Given, $y = 5\sec x + 4\cos x$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= 5 \cdot \frac{d}{dx}(\sec x) + 4 \cdot \frac{d}{dx} \cos x \\ &= 5\sec x \cdot \tan x - 4\sin x \quad \text{Ans}\end{aligned}$$

22. Given, $y = x^{-4}(3-4x^{-5})$

$$y = 3x^{-4} - 4x^{-9}$$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= 3 \cdot \frac{d}{dx} x^{-4} - 4 \cdot \frac{d}{dx} x^{-9} \\ &= 3 \cdot (-4x^{-5}) - 4 \cdot (-9 \cdot x^{-10}) \\ &= -12 \cdot x^{-5} + 36x^{-10} \quad \text{Ans}\end{aligned}$$

23. Given, $y = x^{-3} \cdot (5+3x)$

$$y = 5x^{-3} + 3x^{-2}$$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= 5 \cdot \frac{d}{dx} x^{-3} + 3 \cdot \frac{d}{dx} x^{-2} \\ &= 5 \cdot (-3x^{-4}) + 3 \cdot (-2x^{-3}) \\ &= -15x^{-4} - 6x^{-3} \quad \text{Ans}\end{aligned}$$

24. Given, $y = x^5 \cdot (3-6x^{-9})$

$$y = 3x^5 - 6x^{-4}$$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= 3 \frac{d}{dx} x^5 - 6 \frac{d}{dx} x^{-4} \\ &= 3 \cdot (5x^4) - 6(-4x^{-5}) \\ &= 15x^4 + 24x^{-5} \quad \text{Ans}\end{aligned}$$

25. Given, $y = 2x - \frac{3}{4}$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= 2 \cdot \frac{dx}{dx} - \frac{d}{dx} \left(\frac{3}{4} \right) \\ &= 2 \cdot (1) - 0 \\ &= 2 \quad \text{Ans}\end{aligned}$$

26. Given, $y = 5\sin x - 6\cos x + 7$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= 5 \frac{d}{dx} \sin x - 6 \cdot \frac{d}{dx} \cos x + \frac{d}{dx} (7) \\ &= 5(\cos x) - 6(-\sin x) + (0) \\ &= 5\cos x + 6\sin x \quad \text{Ans}\end{aligned}$$

27. Given, $y=2\tan x-7\sec x$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= 2 \cdot \frac{d}{dx} \tan x - 7 \cdot \frac{d}{dx} \sec x \\ &= 2 \cdot (\sec^2 x) - 7 \cdot (\sec x \cdot \tan x) \\ &= \sec x \cdot (2\sec x - 7\tan x) \quad \text{Ans}\end{aligned}$$

28. Given, $y=x+\frac{1}{x}$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d(x)}{dx} + \frac{d}{dx} \left(\frac{1}{x} \right) \\ &= (1) + \left(\frac{-1}{x^2} \right) \\ &= 1 - \frac{1}{x^2} \quad \text{Ans}\end{aligned}$$

29. Given, $y = \frac{x^2 + 2x + 1}{x}$
 $y = x + 2 + \frac{1}{x}$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d(x)}{dx} + \frac{d(2)}{dx} + \frac{d}{dx} \left(\frac{1}{x} \right) \\ &= (1) + (0) + \left(\frac{-1}{x^2} \right) \\ &= 1 - \frac{1}{x^2} \quad \text{Ans}\end{aligned}$$

30. Given, $y=4\sqrt{x}-2$

d.w.r. to x

$$\begin{aligned}\frac{dy}{dx} &= 4 \frac{d}{dx} \sqrt{x} - \frac{d(2)}{dx} \\ &= 4 \left(\frac{1}{2\sqrt{x}} \right) - (0) \\ &= \frac{2}{\sqrt{x}} \quad \text{Ans}\end{aligned}$$

Long Answer Type Questions (दीर्घ उत्तरीय प्रश्न)

Find the derivative of the following functions by First principal.

निम्नलिखित फलनों का प्रथम सिद्धांत से अवकलज ज्ञात कीजिए।

- | | |
|------------------|-----------------------|
| 1. $f(x)=\sin x$ | 5. $f(x)=e^x$ |
| 2. $f(x)=x$ | 6. $f(x)=x^2$ |
| 3. $f(x)=3$ | 7. $f(x)=\frac{1}{x}$ |
| 4. $f(x)=10x$ | 8. $f(x)=\tan x$ |

Long Answer Type Questions (दीर्घ उत्तरीय प्रश्न)

1. Given, $y=f(x)=\sin x$ ----- (1)

By First principal प्रथम सिद्धांत से

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}, \quad \text{form} = \frac{0}{0} \\ &= \lim_{h \rightarrow 0} \frac{2\cos\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right)}{h} \\ &= \lim_{h \rightarrow 0} \cos\left(x + \frac{h}{2}\right) \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \right) \\ &= \cos\left(x + \frac{0}{2}\right) \cdot (1) \\ &= \cos x \quad \text{Ans}\end{aligned}$$

2. Given, $y=f(x)=x$ ----- (1)

By First principal

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h) - x}{h} \\ &= \lim_{h \rightarrow 0} \frac{h}{h} \\ &= 1 \quad \text{Ans}\end{aligned}$$

3. Given, $y=f(x)=3$ ----- (1)

By First principal

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3-3}{h} \\ &= 0 \quad \text{Ans}\end{aligned}$$

4. Given, $y=f(x)=10x$ -----(1)

By First principal

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{10(x+h) - 10x}{h} \\ &= \lim_{h \rightarrow 0} \frac{10h}{h} \\ &= 10 \quad \text{Ans}\end{aligned}$$

5. Given, $y=f(x)=e^x$ ----- (1)

By First principal

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} \\ &= \lim_{h \rightarrow 0} \left(\frac{e^h - 1}{h} \right) \cdot e^x \\ &= 1 \cdot e^x \\ &= e^x \quad \text{Ans}\end{aligned}$$

6. Given, $y=f(x) = x^2$ -----(1)

By First principal

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2x + h)}{h} \\ &= \lim_{h \rightarrow 0} (2x + h) = 2x \quad \text{Ans}\end{aligned}$$

7. Given, $y=f(x)=\frac{1}{x}$ -----(1)

By First principal

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x - x - h}{h(x+h)x} \\ &= \lim_{h \rightarrow 0} \frac{-h}{h(x+h)x} \\ &= \lim_{h \rightarrow 0} \frac{-1}{(x+h)x} \\ &= \frac{-1}{(x+0)x} \\ &= \frac{-1}{x^2} \quad \text{Ans}\end{aligned}$$

8. Given, $y=f(x)=\tan x$ -----(1)

By First principal

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\tan(x+h) - \tan x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x+h) \cdot \cos x - \sin x \cdot (\cos(x+h))}{h \cdot \cos(x+h) \cdot \cos x} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x+h-x)}{h \cdot \cos(x+h) \cdot \cos x} \\ &= \lim_{h \rightarrow 0} \frac{\sinh}{h \cdot \cos(x+h) \cdot \cos x} \\ &= \lim_{h \rightarrow 0} \left(\frac{\sinh}{h} \right) \times \frac{1}{\cos(x+h) \cdot \cos x} \\ &= (1) \times \frac{1}{\cos(x+0) \cdot \cos x} \\ &= \frac{1}{\cos x \cdot \cos x} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \quad \text{Ans}\end{aligned}$$